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SHEAR STRESS AND HEAT TRANSFER AT A STAGNATION POINT

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Abstract—Experiments have been carried out to determine the shear stress under a free jet impinging on an impervious surface. The measurements indicate that a minimum occurs at the stagnation point with local maxima adjacent and in close proximity. The failure of the data to indicate a zero stress level is attributed to resolution of the probe. The results are related to the situation of heat transfer in an equivalent flow although the direct application of the simple Reynolds analogy was not possible in this complex case.

NOMENCLATURE

c_τ,	local shear stress coefficient, $= \tau/\frac{1}{2}\rho W_0^2$;
h,	jet slot width;
Nu,	Nusselt number;
Pr,	Prandtl number;
p,	pressure;
Re,	Reynolds number;
U,	velocity component along surface;
U_m,	local maximum velocity;
W,	free jet velocity component;
W_m,	local maximum velocity;
W_0,	free jet velocity in nozzle;
x,	coordinate parallel to stagnation surface;
x_0,	half-velocity width of jet;
z,	coordinate parallel to nozzle centerline;
z_0,	separation distance from jet nozzle to stagnation surface;
z_p,	end of potential core for jet.

Greek symbols

δ,	location of U_m;
ρ,	mass density;
τ,	wall shear stress.

INTRODUCTION

THE IMPINGEMENT of a free jet upon a surface is a problem of special interest because of the applications in cooling and drying. In the literature [1] the flow is generally separated into the regimes shown in Fig. 1, i.e. the approaching jet flow, the deflection zone and the wall jet region. The deflection zone is the least understood, particularly the small zone in the immediate vicinity of the stagnation point. In this paper we present some measurements of surface shear stress which have been made in connection with a general study of impinging flows upon stationary and moving surfaces. Some interesting results have been found which, when interpreted in terms of heat transfer, are at variance with published data [3, 4]. In particular, the analysis indicates that a local minimum value exists at the point of stagnation in contrast to the maximum reported by Gardon *et al.* This difference can be attributed to the influence of probe size. A second result indicates that the average heat transfer in the stagnation zone increases monotonically as the separation distance between the impinging jet and the surface is reduced.

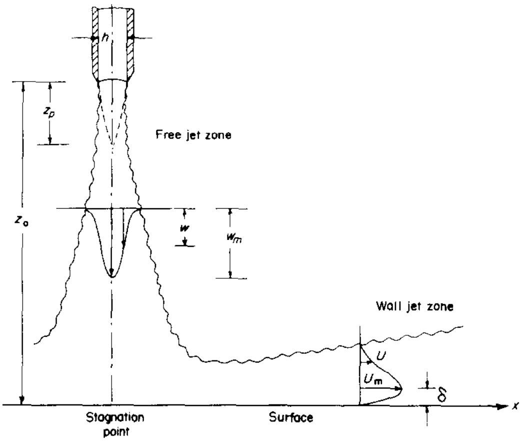

FIG. 1. The impinging jet.

GENERAL DESCRIPTION

The experiment was conducted with a fully turbulent two-dimensional jet impinging normally upon a fixed, slightly curved surface. The distance between the jet exit and the surface z_0, is variable but is always much smaller than the radius of curvature of the surface. This ensured that normal accelerations due to curvature were of a sufficiently small order to be neglected. The free jet velocity field decays according to the relation

$$W_m \sim z^{-\frac{1}{2}} \tag{1}$$

in the established zone beyond the potential core. As well, it spreads linearly as

$$x_0 \sim z \tag{2}$$

where x_0 is the width between points of half-maximum velocity at a section z. Equations (1) and (2) are the accurate self-preserving relations and would be expected to apply only for $z/h > 40$ [4] but for mean velocity these variations were obtained for $z/h > 10$.

When a surface is interposed in the jet a pressure field and strong streamline curvature are produced which extend approximately $x_0/2$ laterally and back into the jet flow. The lateral flow continues as a wall jet of characteristic velocity and half width equal to that of the jet at impingement. Within the impingement region the flow is complex but the stagnation point solution of Hiemenz [5] must apply within a small sub-region because the velocity approaches zero at the stagnation point.

EXPERIMENTAL CONSIDERATIONS

The investigation was carried out in the Fluid Mechanics Laboratories of the Department of mechanical Engineering, University of Toronto. The experimental equipment which is shown schematically in Fig. 2 consisted of a jet assembly mounted within an *A*-frame and directed upon a large cylindrical roll. The slot width was machined precisely to 1·59 mm and the length of 508 mm was sufficient to ensure two-dimensional flow in the central portion. A subsequent check of the velocity field indicated that the impinging flow was uniform along the jet over the central eighty per cent of the roll.

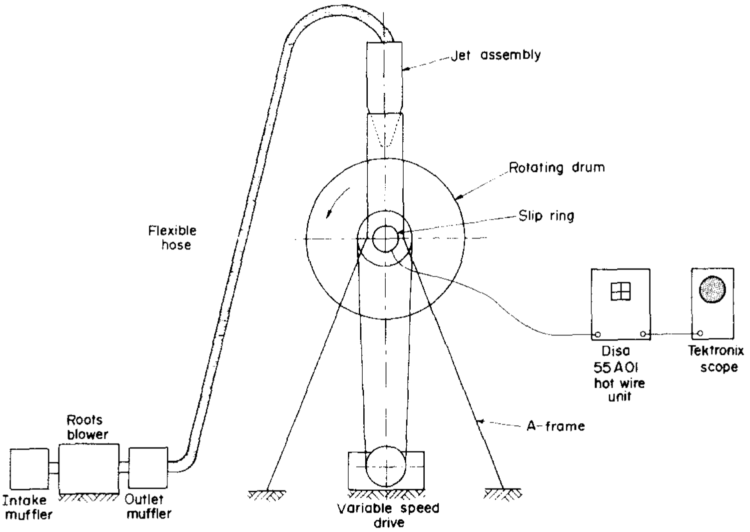

FIG. 2. Experimental apparatus.

The air for the jet was supplied by a "Roots" rotary-lobe blower. This was led into a plenum chamber above the jet nozzle. Within the chamber a honey-comb grid and screen combination straightened the flow and distributed it uniformly to the converging nozzle slot.

Mean velocity measurements were made with both pitotstatic tubes and conventional hot-wires. A constant-temperature anemometer system, DISA model 55A10, was used with platinum–tungsten wires, 1·2 mm long and 5 μm in diameter. The wires were calibrated directly in the jet flow and the signal linearized.

The measurement of surface shear stress is rather more difficult and two schemes were used. In the first, a surface film probe, DISA model 55A92, was mounted flush with the drum surface. The probe consisted of a platinum film 0·20 mm wide and 1 mm long supported upon the end of a cylinder approximately 2 mm in diameter. The calibration of the hot-film was performed in the fully developed turbulent flow in a circular pipe. In this situation, the shear stress can be determined from a measurement of the pressure drop using the Darcy Weisbach equation. The maximum velocity in the pipe, however, was not high enough to duplicate the shear stress levels encountered on the roll surface. An extrapolation procedure was therefore devised to extend the calibration curve. This was consistent with the known performance of surface hot-film probes [5].

In the process of extending the investigation to the case of a moving surface it was found that the surface hot-film probe suffered a lack of dynamic response. Another technique based on a hot-wire probe was thus devised. It was used in this present series of tests to investigate the stagnation zone region. The calibrated hot wire was mounted from the inside of the roll such that only the tips of the prongs extended above the surface. The wire, 8 μm in diameter was mounted parallel to the surface with the centreline 25 μm above it. This was found to be superior in operation to the hot film but the calibration in terms of shear stress depended on an assumption of a linear profile between the wire and the surface, which was subsequently verified.

RESULTS

The mean velocity at the nozzle exit was measured along the length of the roll to check the uniformity of the flow. For an average velocity of 53 m/s the variation was less than 2 per cent in the uniform central region which was felt acceptable. With the roll removed, mean velocity profiles of the jet were taken at a number of stations. These are plotted in Fig. 3, non-dimensionalized by the local maximum velocity W_m and the half-velocity width x_0. As expected the profiles are self-

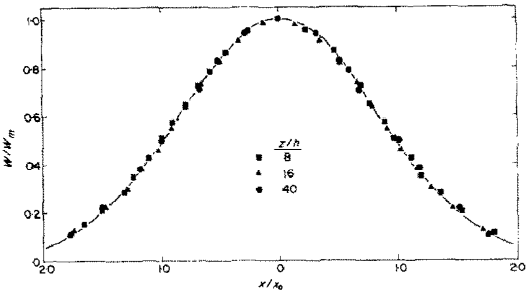

FIG. 3. Velocity profiles for the free

similar. Data for the stations closest to the nozzle are not included since the flow in this region is not fully established and retains characteristics of the potential core. In Fig. 4 the variation of the local scales is plotted. For $z/h > 10$, the flow is seen to be self-preserving, the centreline velocity and spread varying as in equations (1) and (2). From the exit to $z/h = 4$ the maximum velocity is constant, i.e. within the potential core. The value of W_m/W_0 is greater than unity because the velocity profile is not uniform and W_0 was defined as the average velocity at the jet exit. The results are otherwise consistent with those observed previously [3, 6].

The shear stress at the surface of the drum was determined for constant values of z_0/h from 2·4 to 48·0. The resulting profiles determined with the hot film surface probe are shown in Fig. 4. The data were obtained by indexing the roll, thus changing the position of the probe with respect to the impingement point. It was found that these data were consistently repeatable. For $x/h > 10$ the stress decreases monotonically as under a simple wall jet. An interesting feature of the profiles is a pair of secondary maxima occurring at $x/h = 5$ for $z/h = 2$. This effect was also noted by Gardon and Akfirat [4] who interpreted it as a transition from laminar to turbulent boundary layer flow along the surface. This explanation is plausible because there is no other change in the flow region.

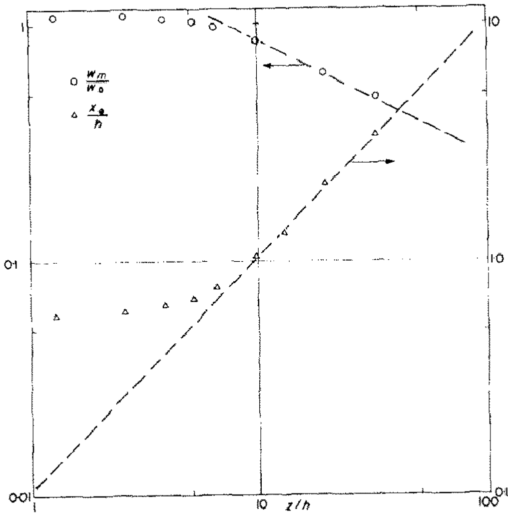

FIG. 4. Streamwise variation of local scales for the free jet.

Each of the curves in Fig. 5 displays a minimum at the stagnation point and local maxima within approximately 2 nozzle widths. The existence of this minimum is to be expected because of symmetry. The shear stress must change sign at the stagnation point. The measured minimum is not close to zero, however, and this must be attributed to inaccuracies in the technique. The finite size of the surface probe will average the shear stress over the width of the sensing element and this produces a non-zero value of stress.

Similar measurements made with the surface hot-wire, bear out this conclusion. Figure 6 shows the dimensionless shear stress for a small zone in the neighbourhood of impingement. The approach to zero at the stagnation point is more pronounced as would be expected with the relatively smaller size of the wire. It is noted that the shear stress varies linearly in this region, as predicted by the laminar stagnation point flow [7]. The agreement between the two sets is reasonable for $x/h > 1$ where the stress gradient is not large. These results suggest that in the immediate vicinity of the stagnation point the hot-wire probe will provide superior resolution although for the major portion of the shear stress profile, the surface hot-film is adequate.

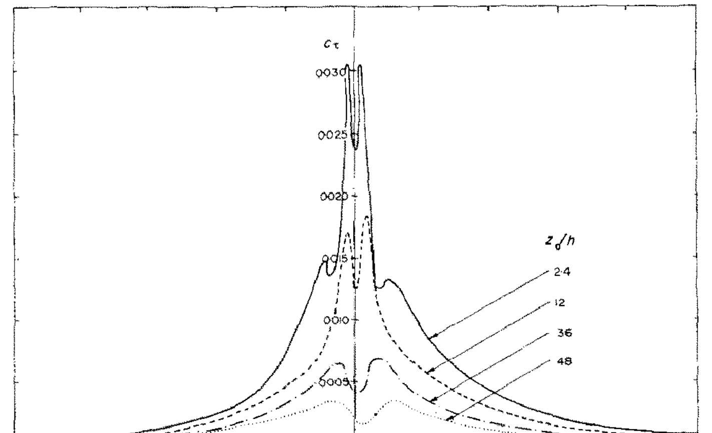

FIG. 5. Shear stress as determined by the hot-film surface probe.

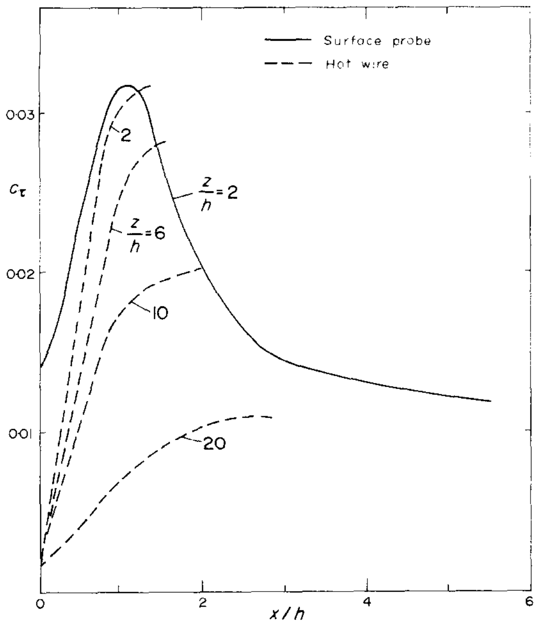

FIG. 6. Shear stress in the stagnation region.

HEAT-TRANSFER COEFFICIENT

The impingement of a two-dimensional jet on a solid surface is a boundary-layer flow. The streamlines for the fluid next to wall are parallel to it except in the immediate vicinity of the stagnation point. Furthermore, velocity gradients of appreciable magnitude exist only in a thin layer along the wall. For all established boundary-layer flows the general similarity between the transfer of heat and momentum can be expressed by the Reynolds analogy which, when written in terms of the flow parameters based on local velocity, length and temperature scales is

$$Nu = \tfrac{1}{2}C_\tau Re\, Pr^{\frac{1}{3}}. \qquad (3)$$

Because of its wide applicability it is tempting to apply equation (3) to this flow using the assumption that the local scales are equal to the boundary scales. This assumption is valid for flows in pipes and along flat plates. Figure 7 presents the predicted Nusselt number along with the measurements of heat transfer by Gardon and Akfirat [4] expressed in dimensionless form. Also included are the mass-transfer measurements of Korger and Krizek [8] which have been corrected to a Schmidt number = 0·72 to be directly comparable to the heat transfer with air as the fluid. It is seen that the agreement between the heat- and mass-transfer measurements is very good and thus one concludes that the values are accurate. However, the values predicted from equation (3) are considerably larger in the region $x/h < 5$ and moderately larger for $x/h > 5$. It must be concluded that the jet properties, h and W_0 used in equation (3) do not describe the local width and velocity scales. Another prediction of Nu from C_τ using equation (3) was made using the measured local maximum velocity and width but this, too, did not agree with the direct measurements. It must be concluded that a simple Reynolds analogy does not exist for this rapidly developing turbulent flow. This is reinforced by the observation that the wall jet does not reach its equilibrium form for $x/h < 25$, a value well beyond the limit of this study. The data of Myers, Schauer and Eustis [11, 12] shows that there is a very long development length for a wall jet and that a simple form for equation (3) cannot be written for $x/h < 25$.

At the stagnation point the flow is described by the classic solution of Hiemenz [7]. The boundary-layer thickness is constant, the shear stress increases linearly with distance and the Nusselt number must be constant. The magnitude of the Nusselt number can be readily determined using Eckert's solution [9] but this contains the constant from the potential stagnation point solution. This constant is difficult to determine by experiment and cannot be related to the velocity and size of the nozzle. It is, however, related to the shear gradient in the Hiemenz solution [7]. In effect, the Nusselt number is determined by the fluid viscosity and the measured shear gradient. By following this procedure stagnation point values were predicted from hot-wire data and are plotted in Fig. 8 as a function of separation distance. These are compared to the measurements of Gardon and Akfirat which appear to be considerably higher than the results obtained from the gradient of the shear stress. Here discrepancy may lie with the heat-transfer measurements. The meter used by Gardon and Akfirat was significantly larger than the hot-wire probe and hence averaged the flux over a greater lateral extent. If our present data were averaged over a comparable distance, the effect of this decreased resolution would be to increase the level of shear stress recorded by the probe bringing it more in line with the heat-transfer data. In fact, a few of the profiles published by Gardon and Akfirat [4] show some evidence of a local minimum at the stagnation point.

An analysis, applicable to any boundary type flow has recently been presented by Davies [10]. He applied it to an axisymmetrical jet impinging upon a plane surface. This has been modified for the present two-dimensional flow with the result shown in Fig. 8. It is seen that the Davies prediction is close to our results although this may be fortuitous considering the accuracy of the assumptions. It is significant, however, that the Davies curve is a monotonically decreasing function with increasing separation distance. This is of particular significance in practical heat-transfer situations. For large heat transfer the jet must be located as close to the surface as possible.

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REFERENCES

1. H. Hardisty, Industrial drying using impinging air jets, Rep. No. 226, School of Engineering, Univ. of Bath (1972).

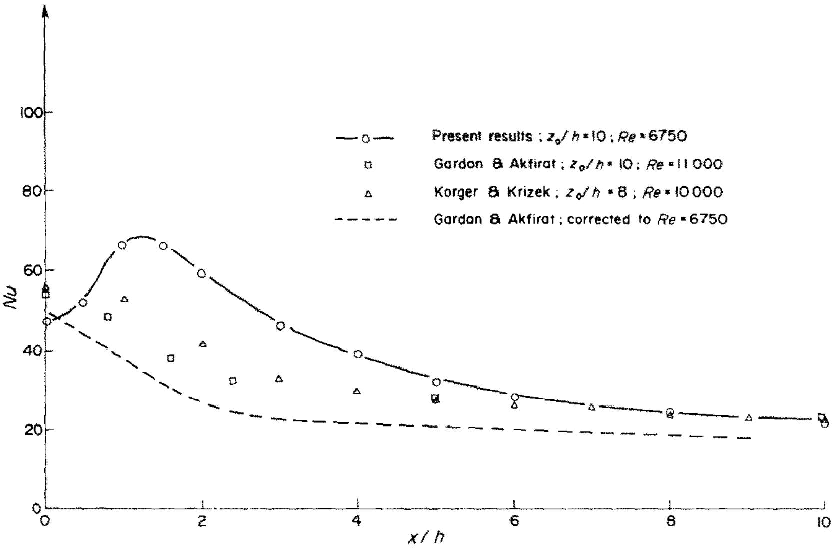

FIG. 7. Nusselt number predicted from simple Reynolds analogy.

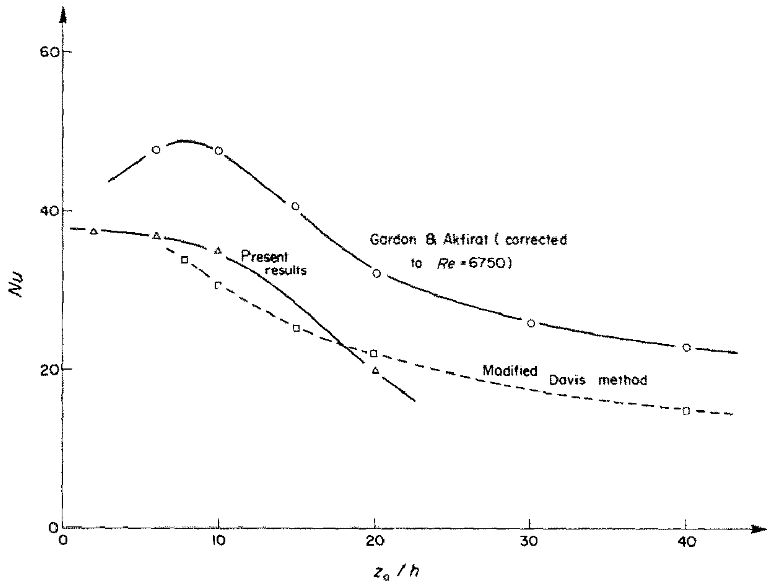

FIG. 8. Heat transfer at stagnation point.

2. R. Gardon and J. Cobonque, Heat transfer between a flat plate and jets of air impinging on it, *International Developments in Heat Transfer*, pp. 454–460. A.S.M.E., New York (1962).
3. R. Gardon and J. C. Akfirat, The role of turbulence in determining the heat transfer characteristics of impinging jets, *Int. J. Heat Mass Transfer* **8**, 1261–1271 (1965).
4. R. Gardon and J. C. Akfirat, Heat transfer characteristics of impinging two-dimensional air jets, *J. Heat Transfer* **88**, 101–108 (1966).
5. B. J. Bellhouse and P. J. Schultz, Determination of mean and dynamic skin friction separation and transition in low speed flow with a thin film heated element, *J. Fluid Mech.* **24**, 379–390 (1966).
6. G. Heskestad, Hot-wire measurements in a plane turbulent jet, *J. Appl. Mech.* **32**, 721–734 (1965).
7. K. Hiemenz, *Boundary Layer Theory*, edited by H. Schlichting, 6th edn, p. 87. McGraw-Hill, New York (1968).
8. M. Korger and F. Krizek, Mass transfer coefficient in impingement flow from slotted nozzles, *Int. J. Heat Mass Transfer* **9**, 337–344 (1966).
9. E. R. G. Eckert, Die Berechnung des Wärmeüberganges in der laminaren grenzschicht umströmter Körper, *VDI-Forchungsheft* **416**, 1–24 (1942).
10. J. T. Davies, *Turbulence Phenomena*. Academic Press, New York (1972).
11. G. E. Myers, J. J. Schauer and R. H. Eustis, Plane turbulent wall jet flow development and friction factor, *J. Basic Engng* **85**, 47–54 (1963).
12. G. E. Myers, J. J. Schauer and R. H. Eustis, Heat transfer to plane turbulent wall jets, *J. Heat Transfer* **85**, 209–214 (1963).

TENSION DE CISAILLEMENT ET TRANSFERT DE CHALEUR EN UN POINT D'ARRET

Résumé—Des expériences ont été effectuées afin de déterminer la tension de cisaillement sous un jet libre frappant une paroi imperméable. Les mesures montrent qu'un minimum a lieu au point d'arrêt avec un maximum local immédiat situé à une distance très proche. L'impossibilité de trouver expérimentalement un niveau nul de tension de cisaillement est attribuée au pouvoir de résolution de la sonde. Les résultats sont rapprochés des conditions de transfert thermique dans un écoulement semblable quoique l'application directe de la simple analogie de Reynolds n'est pas possible dans ce cas complexe.

SCHUBSPANNUNG UND WÄRMEÜBERGANG AN EINEM STAUPUNKT

Zusammenfassung—Die Wandschubspannung eines auf eine undurchlässige Oberfläche auftreffenden Freistrahls wurde experimentell ermittelt. Die Messungen zeigen, daß am Staupunkt ein Minimum auftritt mit in unmittelberer Nähe auftretenden Maxima. Der Umstand, daß der Wert null für die Schubspannung nicht gemessen werden konnte, wird auf die begrenzte Auflösung der Sonde zurückgeführt. Die Ergebnisse werden mit dem Wärmeübergang einer äquivalenten Strömung in Beziehung gebracht, obwohl die Anwendung der einfachen Reynolds-Analogie in diesem komplexen Fall nicht möglich ist.

КАСАТЕЛЬНОЕ НАПРЯЖЕНИЕ И ТЕПЛОПЕРЕНОС В КРИТИЧЕСКОЙ ТОЧКЕ

Аннотация — Проводились эксперименты для определения касательного напряжения в условиях свободного падения струи на непроницаемую поверхность. В результате измерений оказалось, что минимум касательных напряжений имеет место в критической точке. Максимальные значения локальных величин касательных напряжений имеют место вблизи критической точки. Отсутствие данных по нулевому уровню напряжения трения объясняется разрешающей способностью датчика. Результаты отнесены к случаю теплопереноса в эквивалентном потоке, хотя непосредственное применение простой аналогии Рейнольдса оказалось невозможным в этом сложном случае.